

# The Origins of Blue Stragglers and Binarity in Globular Clusters

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## ABSTRACT

Two basic formation channels have been proposed for blue straggler stars in globular clusters: binary star evolution and stellar collisions. We recently showed that the number of blue stragglers found in the core of a globular cluster is strongly correlated with the total stellar mass of the core, but not with the collision rate in the core. This result strongly favoured binary evolution as the dominant channel for blue straggler formation. Here, we use newly available empirical binary fractions for globular clusters to carry out a more direct test of the binary evolution hypothesis, but also of collisional channels that involve binary stars. More specifically, using the correlation between blue straggler numbers and core mass as a benchmark, we test for correlations with the number of binary stars, as well as with the rates of single-single, single-binary, and binary-binary encounters. We also consider joint models, in which blue straggler numbers are allowed to depend on star/binary numbers and collision rates simultaneously.

Surprisingly, we find that the simple correlation with core mass remains by far the strongest predictor of blue straggler population size, even in our joint models. This is despite the fact that the binary fractions themselves strongly anti-correlate with core mass, just as expected in the binary evolution model.

At first sight, these results do not fit neatly with either binary evolution or collisional models in their simplest forms. Arguably the simplest and most intriguing possibility to explain this unexpected result is that observational errors on the core binary fractions are larger than the true intrinsic dispersion associated with their dependence on core mass. In the context of the binary evolution model, this would explain why the combination of binary fraction and core mass is a poorer predictor of blue straggler numbers than core mass alone. It would also imply that core mass is a remarkably clean predictor of core binary fractions. This would be of considerable importance for the dynamical evolution of globular clusters, and provides an important benchmark for models attempting to understand their present-day properties.

**Key words:** blue stragglers – binaries: close – globular clusters: general – methods: statistical – stellar dynamics.

## 1 INTRODUCTION

Blue stragglers (BSs) in globular clusters (GCs) are stars that appear brighter and bluer than the main-sequence turn-off (MSTO) in the cluster colour-magnitude diagram (CMD) (Sandage 1953). They are thought to be created

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when fresh hydrogen is mixed into the core of a normal low-mass main-sequence (MS) star (e.g. Sills et al. 2001, 2002). Two main formation channels have been proposed for BSs in GCs: binary star evolution and dynamical interactions. BSs can form via the former pathway either through binary mass-transfer due to Roche-lobe overflow from an evolved primary onto a normal MS companion (McCrea 1964; Geller & Mathieu 2011), or through the coalescence of two normal MS stars in a binary system. The latter mechanism can occur due to angular momentum loss induced by a magnetized stellar wind (e.g. Iben & Tutukov 1984; Andronov, Pinsonneault & Terndrup 2006), or even Kozai cycles induced by an outer triple companion (Perets & Fabrycky 2009). The dynamical pathway involves collisions between two or more MS stars (e.g. Sills & Bailyn 1999). These are typically mediated by dynamical interactions involving binary stars, since the cross-section for collision is much larger for a binary than it is for a single star (e.g. Leonard 1989; Leonard & Linnell 1992).

Several statistical studies have been conducted in search of a dominant BS formation channel. However, the cluster parameter that has thus far yielded the strongest correlation with BS population size is the cluster (Piotto et al. 2004; Leigh, Sills & Knigge 2007) or core (Knigge, Leigh & Sills 2009; Leigh, Sills & Knigge 2011a) mass. Many authors have tried to explain this by simultaneously invoking multiple formation mechanisms. For example, Davies, Piotto & De Angeli (2004) suggested that the observed dependence of BS numbers on total cluster mass can be explained if BSs in low-mass clusters are primarily descended from binaries, whereas BSs in high-mass clusters are primarily descended from collisions. A similar scenario has been argued for in an attempt to explain the bimodal BS radial distribution observed in many globular clusters (e.g. Ferraro et al. 1993, 2004; Mapelli et al. 2006; Lanzoni et al. 2007; Beccari et al. 2011; Sanna et al. 2012). In this picture, BSs in the dense core were formed in collisions, whereas BSs in the low-density cluster outskirts were formed by mass-transfer within primordial binaries.

At the time these studies were conducted, there were hardly any observational constraints on the properties of the binary populations in GCs. In particular, empirical binary fractions were available for only a small subset of low-density GCs (Sollima et al. 2008). This issue was only recently resolved by Milone et al. (2012). Using data from the HST-based ACS Survey of Globular Clusters, these authors derived photometric binary fractions for the MS populations in 59 Milky Way (MW) GCs. This sample offers a long-awaited opportunity to test more directly whether the sizes of BS and binary populations in GCs are correlated, as one might expect for both the binary evolution channel and for collisional formation channels that involve binaries (i.e. 1+2 and 2+2 encounters). This was previously addressed by Sollima et al. (2008) and Milone et al. (2012). These results provided evidence that the blue straggler fraction is indeed related to the binary fraction, and we build on those previous works in this paper.

The plan of this paper is as follows. First, in Section 2, we present the observational data. We then explain and carry out our analysis of these data in Section 3. There, we derive the expected scaling laws for the simplest versions of the various formation channels and compare these theoretic-

cal predictions to the observations. Finally, the implications of our results for the formation and evolution of BSs in GCs are discussed in Section 4.

## 2 OBSERVATIONAL DATA

BS numbers are taken from Table 1 of Leigh, Sills & Knigge (2011a), which was compiled using data taken from the ACS Survey for Globular Clusters (Sarajedini et al. 2007)<sup>1</sup> The sample used in this paper omits five clusters from the catalogue of Leigh, Sills & Knigge (2011a), since we do not have observed binary fractions in these cases. We use only those BS number counts within the core and within four core radii from the cluster centre (columns 4 and 7, respectively, in Table 1 of Leigh, Sills & Knigge (2011a)) in this paper.

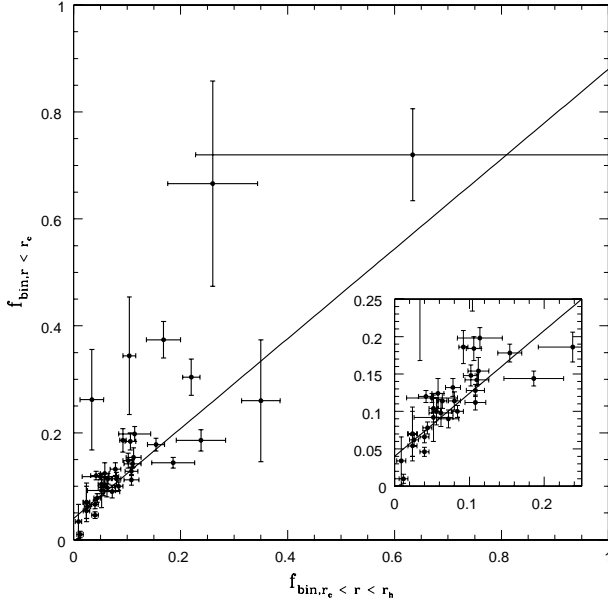
Binary fractions within the core ( $r < r_c$ ;  $f_{bin,core}$ ) and the half-mass radius ( $r < r_h$ ;  $f_{bin,half}$ ) are taken from Table 1 of Milone et al. (2012).<sup>1</sup> The latter values are not provided directly in Milone et al. (2012). Instead, binary fractions within the annulus separating the core and the half-mass radius ( $r_c < r < r_h$ ;  $f_{bin,r_c < r < r_h}$ ) are given. Therefore, we calculate mass-weighted binary fractions within the half-mass radius using the relation:

$$f_{bin,half} = \frac{f_{bin,core}M_{core} + f_{bin,r_c < r < r_h}(M_{half} - M_{core})}{M_{half}}, \quad (1)$$

where  $M_{core}$  and  $M_{half}$  are the mass of the cluster core and the mass contained within the half-mass radius, respectively. In order to obtain accurate estimates for the total stellar mass contained within the core, we generated single-mass King models calculated using the method of Sigurdsson & Phinney (1995) to obtain luminosity density profiles for every cluster in our sample. The profiles were found using the concentration parameters of McLaughlin & van den Marel (2005) and the central luminosity densities of Harris (1996, 2010 update). We then integrated the derived luminosity density profiles numerically in order to estimate the total stellar light contained within the core, which we multiplied by a mass-to-light ratio of 2 in order to obtain estimates for the total stellar mass contained within one core radius from the cluster centre. The mass enclosed within the half-mass radius was then estimated by calculating the total cluster mass from its absolute integrated visual magnitude (once again assuming a mass-to-light ratio of 2), and then dividing by two. Throughout this paper, we have adopted the binary fractions provided in column 6 of Table 1 in Milone et al. (2012), which provides the *total* fraction of objects that are binaries within the indicated annulus. However, in practice, this estimate of the total number of binaries is extrapolated from the observed fraction of binaries with mass ratio  $q > 0.5$  by assuming a flat distribution in  $q$ .

<sup>1</sup> The data can be found at [http://www.astro.ufl.edu/~ata/public\\_hstgc/](http://www.astro.ufl.edu/~ata/public_hstgc/), and was last accessed on 02/02/11.

<sup>1</sup> These binary fractions have been corrected for a variety of observational biases, including completeness, contamination from field stars, and differential reddening. Detailed explanations of these procedures have been provided in Sarajedini et al. (2007), Anderson et al. (2008) and Milone et al. (2012).



**Figure 1.** The binary fraction in the core plotted versus the binary fraction in the annulus separating the core and the half-mass radius. Error bars are taken directly from Table 1 of Milone et al. (2012). The solid line shows the weighted least-squares fit to the data provided in Equation 2.

There are several clusters for which the binary fractions in Milone et al. (2012) are provided for  $f_{bin,r_c < r < r_h}$  but not  $f_{bin,core}$ . To approximate  $f_{bin,core}$  in those clusters for which these values are missing, we perform a weighted least-squares fit to quantify the dependence of  $f_{bin,core}$  on  $f_{bin,r_c < r < r_h}$  using every cluster in the sample of Milone et al. (2012) for which both of these quantities were given. We then supplement  $f_{bin,core}$  in every cluster for which only  $f_{bin,r_c < r < r_h}$  was provided. This is only necessary in a handful of clusters, but the resulting increase in our sample size is nevertheless worth while. We obtain a relation of the form:

$$f_{bin,core} = (0.84 \pm 0.15)f_{bin,r_c < r < r_h} + (0.04 \pm 0.01). \quad (2)$$

This is shown in Figure 1. The fit is good for binary fractions less than  $\sim 0.2$ , but the agreement is poor for larger binary fractions. In order to test the effects had on our least-squares fit by clusters with large binary fractions, we re-perform it considering only those clusters for which both  $f_{bin,core} < 0.2$  and  $f_{bin,r_c < r < r_h} < 0.2$ . In this case, both the slope and y-intercept of the fit agree with those reported in Equation 2 to within one standard deviation. We conclude that the fit and the corresponding uncertainties provided in Equation 2 are reasonable.

Our analysis in Section 3 also requires the absolute visual magnitude ( $M_V$ ), core radius ( $r_c$ ), central luminosity density ( $\rho_0$ ) and central velocity dispersions ( $\sigma_0$ ) of each cluster. All of these quantities are taken from Harris (1996, 2010 update), except for the velocity dispersion of 10 clusters for which these values are not provided. In these cases, we use the calculated velocity dispersions provided by Webbink (1985).

### 3 ANALYSIS AND RESULTS

In this section, we derive theoretical scaling laws for the simplest versions of the various proposed blue straggler formation channels and compare these theoretical predictions to the observational data. In practice, this means we will search for correlations between the number of blue stragglers in the core of each GC and the cluster parameter that should set this number in each scenario.

We assess the significance of our correlations primarily via the Spearman rank test, which provides both a correlation coefficient ( $r_s$ ) and the significance level at which the null hypothesis of zero correlation is disproved ( $p_s$ ). A small  $p_s$ -value is indicative of a significant correlation. In addition, we perform weighted least-squares fits to quantify the dependence of BS numbers on each parameter. Uncertainties for all number counts are obtained assuming Poisson statistics, but we also allow for intrinsic dispersion in our fits, at whatever level is required to achieve a reduced  $\chi^2 \simeq 1$ .

Table 1 shows the results of our comparisons between the observed BS numbers and the parameter we have tested. Each entry in Table 1 gives the slope for the line of best-fit, Spearman correlation coefficient, and probability at which the null hypothesis of zero correlation is disproved. These are provided in the form (slope;  $r_s$ ,  $p_s$ ).

#### 3.1 Core Mass

We begin by revisiting the correlation between core BS numbers and core mass, with the latter estimated from our King model fits. Core mass was found to be the best predictor of BS numbers in the core in Knigge, Leigh & Sills (2009) and Leigh & Sills (2011b). Our working assumption in those studies was that core mass was able to predict BS numbers because it is a partial proxy for the (unknown) number of binaries in the core (since  $N_{bin,core} = f_{bin,core}M_{core}$ ; see Section 3.2). Now that binary fractions are available, we can test this assumption directly. However, the original correlation with just core mass remains a useful benchmark in this context: if the empirical binary fractions have added useful information, including them should allow us to discover even stronger correlations.

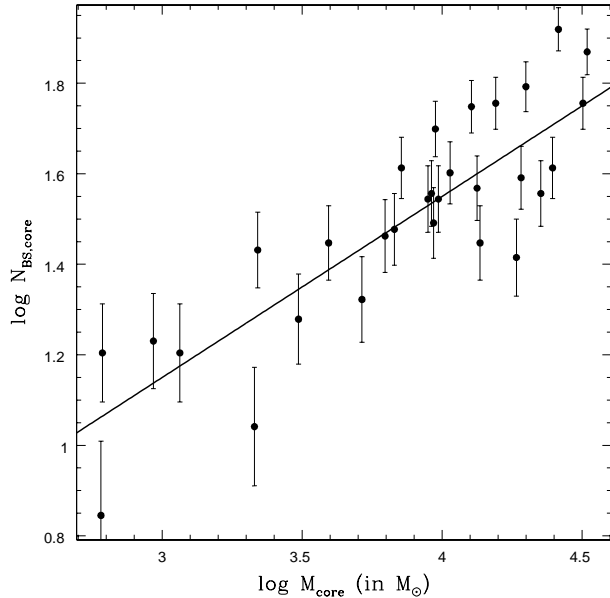
We find a dependence  $N_{BS,core} \propto M_{core}^{0.40 \pm 0.05}$ , and a Spearman correlation coefficient 0.83. This correlation is shown in Figure 2, along with the corresponding line of best-fit to the data. The power-law index on  $M_{core}$  is inconsistent with zero at the  $8\sigma$  confidence level, but also inconsistent with unity at the  $12\sigma$  confidence level. Therefore, the strong, sub-linear correlation between  $N_{BS,core}$  and  $M_{core}$  first reported in Knigge, Leigh & Sills (2009) is confirmed in the present analysis.

#### 3.2 Binary Population Size

If most of the BSs in our sample are descended from binary evolution, we predict a dependence of the form:

$$N_{BS,core} \propto N_{bin,r} \sim \frac{f_{bin,r} M_{encl,r}}{\bar{m}}, \quad (3)$$

where  $N_{BS}$  is the number of BSs within a given radius  $r$  from the cluster centre,  $N_{bin,r}$  is the number of binaries contained within  $r$ ,  $f_{bin,r}$  is the fraction of objects within  $r$  that are



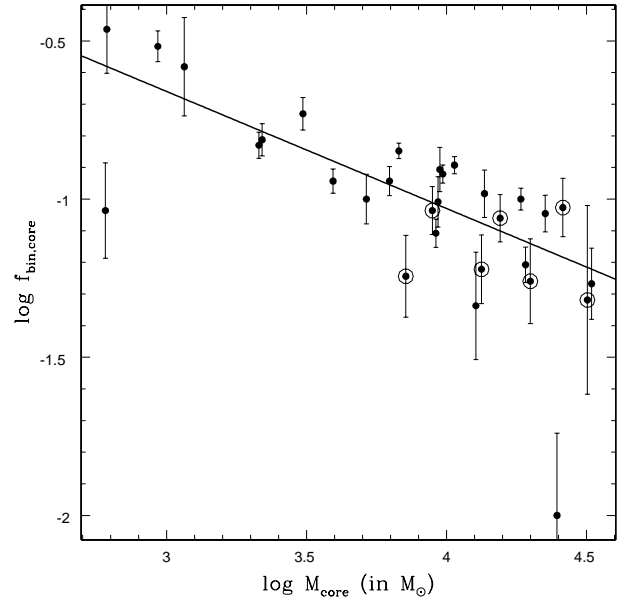
**Figure 2.** The logarithm of the number of BSs as a function of the core mass (in units of solar masses). The solid line shows the best-fit to the data.

binaries,  $M_{encl,r}$  is the total stellar mass enclosed within  $r$ , and  $\bar{m}$  is the average stellar mass (for which we assume the same value in all clusters).

As pointed out in Knigge, Leigh & Sills (2009), Equation 3 allows a simple explanation for the observed sub-linear correlation between  $N_{BS,core}$  and  $M_{core}$ . If we identify  $M_{core}$  with  $M_{encl,r}$  (i.e. we take the relevant radius to be  $r = r_{core}$ ), then  $N_{BS,core} \propto M_{core}^{0.4}$  follows from Equation 3 if the core binary fraction is itself a function of core mass, i.e.  $f_{bin,core} \propto M_{core}^{-0.6}$ . We also showed in Knigge, Leigh & Sills (2009) that there was indeed some evidence for an anti-correlation between core mass and core binary frequency, based on the data set of Sollima et al. (2008); this is a preliminary version of the ACS-based data set we use here.

The final ACS binary fraction data set available now (Milone et al. 2012) allows us to check whether the evidence for this anti-correlation holds up. Figure 3 shows that it does. More specifically, we find  $f_{bin,core} \propto M_{core}^{-0.37 \pm 0.06}$ , while the Spearman correlation coefficient is  $r_s = -0.72$ . The slope for this anti-correlation is not quite as steep as in our naive prediction, but the existence of the anti-correlation itself is obviously promising.

However, this promise is not actually borne out. When we compare  $N_{BS,core}$  directly to  $N_{bin,core} = f_{bin,core} M_{core}$  (Figure 4; Table 1), we find that the *strength* of the correlation actually *decreases* compared to the correlation with just  $M_{core}$  (the Spearman rank coefficient drops from 0.83 to 0.61), while the best-fit *slope* increases only marginally (from  $0.40 \pm 0.05$  to  $0.48 \pm 0.09$ ). Increasing the size of the region considered does not improve things: when we compare the number of BSs within four core radii from the cluster centre against the number of binaries within the half-mass radius, we still find only a weak correlation with a clearly sub-linear slope (Figure 5; Table 1). This is a surprising re-



**Figure 3.** The logarithm of the core binary fraction as a function of the core mass (in units of solar masses). The solid line shows the best-fit to the data. The seven data points with open circles around them correspond to the core binary fractions calculated using Equation 2.

sult at first sight, and we will discuss its implications further in Section 4.

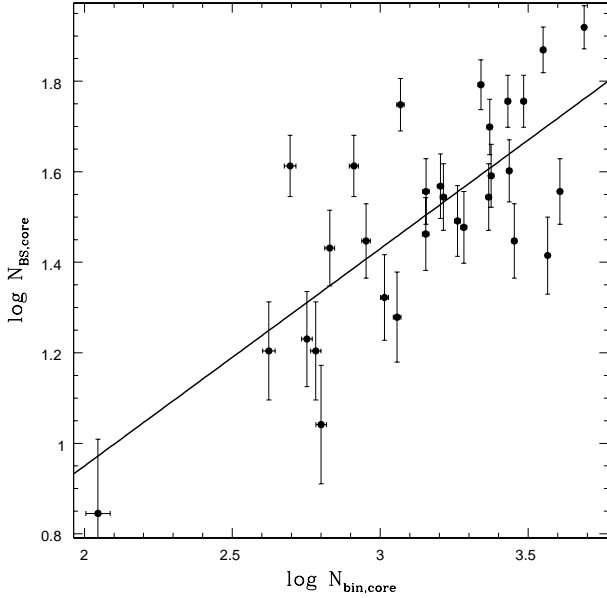
Before moving on, it is worth stressing that the strong anti-correlation between binary fraction and core mass shown in Figure 3 is interesting in its own right. Empirically, its existence is in line with a similarly strong correlation between core binary fraction and cluster absolute magnitude that was already presented in Milone et al. (2012)). This is because absolute magnitude is a proxy for total cluster mass, which in turn correlates strongly with core mass among GCs. Theoretically, however, it is far from clear why abundance of binary stars should depend so strongly on either the core or the total stellar mass of their host clusters. The fact that it does must be important for our understanding of cluster dynamics. Presumably, dynamical evolution is responsible for establishing this correlation and is, in turn, affected by it.

### 3.3 Collision Rates

If most of the BSs in our sample were formed from direct collisions between single stars, then we predict a dependence of the form:

$$N_{BS} \propto N_{1+1} \sim \int_{\tau_0}^{\tau_{cl}} \Gamma_{1+1} dt, \quad (4)$$

where  $\Gamma_{1+1} = 1/\tau_{1+1}$  is the rate of single-single collisions producing BSs (we use the form given in Leigh & Sills (2011b)), and we are integrating with respect to time. We use the age of the cluster ( $\tau_{cl}$ ) as the upper limit of integration, and the average BS age ( $\tau_{BS}$ ) to calculate the lower limit of integration according to  $\tau_0 = \tau_{cl} - \tau_{BS}$ . Assuming  $\Gamma_{1+1}$  remains constant in time (Leigh, Sills & Knigge



**Figure 4.** The logarithm of the number of BSs (within the core only) as a function of the number of binary stars in the core. The solid line shows the best-fit to the data (the slope for which is shown in Table 1).

2011c), Equation 4 simplifies to:

$$N_{BS} \propto N_{1+1} \sim \Gamma_{1+1} \tau_{BS}. \quad (5)$$

If most of the BSs in our sample were formed from dynamical interactions involving binaries, then we predict either a relation of the form:

$$N_{BS} \propto N_{1+2} \sim \Gamma_{1+2} \tau_{BS}, \quad (6)$$

where  $\Gamma_{1+2} = 1/\tau_{1+2}$  is the rate of single-binary encounters, or:

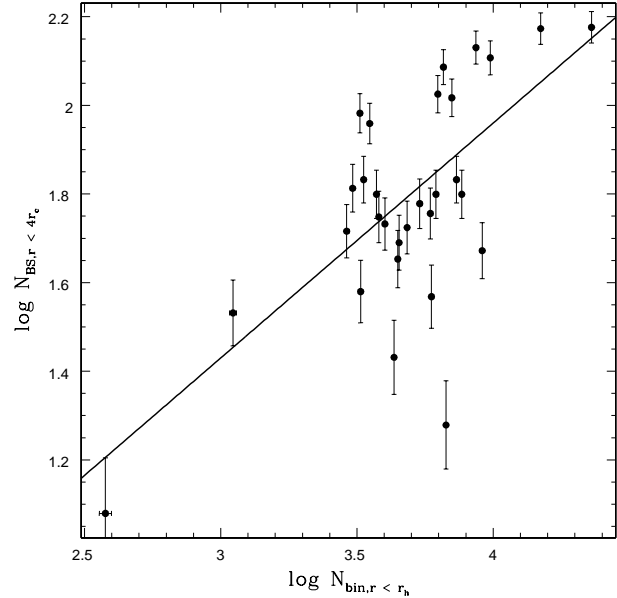
$$N_{BS} \propto N_{2+2} \sim \Gamma_{2+2} \tau_{BS}, \quad (7)$$

where  $\Gamma_{2+2} = 1/\tau_{2+2}$  is the rate of binary-binary encounters.

We find a correlation between the number of BSs in the core and the 1+1 collision rate. The slope for this relation is inconsistent with zero at nearly the  $4\sigma$  confidence level, and it yields a Spearman correlation coefficient that is only slightly smaller than we find for the numbers of binaries in the core. For the 1+2 and especially the 2+2 collision rates, however, our results are consistent with little to no correlation with BS population size. The relations for the 1+1, 1+2, and 2+2 collision rates are plotted in Figure 6, Figure 7, and Figure 8, respectively, along with the corresponding lines of best-fit given in Table 1.

### 3.4 Joint Models

Several authors have suggested that multiple mechanisms could contribute significantly to BS formation. In particular, different formation mechanisms could operate simultaneously *within the same cluster* (e.g. Ferraro et al. 2004), and/or different formation mechanisms could dominate *in different clusters* (e.g. Davies, Piotto & De Angeli 2004).



**Figure 5.** The logarithm of the number of BSs (within four core radii from the cluster centre) as a function of the number of binary stars within the half-mass radius. The solid line shows the best-fit to the data.

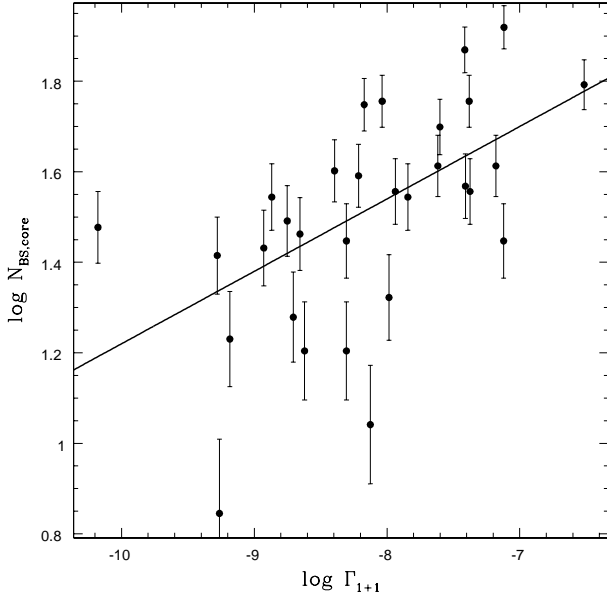
One prediction that is consistent with the second scenario is that the BSs in the high-density clusters in our sample were formed from collisions, whereas the BSs in the low-density clusters were formed from binary star evolution. The first scenario, on the other hand, predicts that some linear combination of the parameters  $N_{bin}$ ,  $N_{1+1}$ ,  $N_{1+2}$ , and  $N_{2+2}$  should yield the strongest correlation with observed BS numbers.

We perform two additional comparisons in an effort to test the idea that multiple formation mechanisms contributed to the formation of the BSs in our sample. First, we divide our sample into low- ( $\log \rho_0 < 3.3$ ) and high-density ( $\log \rho_0 > 3.3$ ) sub-samples of roughly equal size, and independently re-perform our analysis on each. This comparison will tell us whether different formation mechanisms dominate in each of these sub-samples independently. We use the cluster density to divide our sample since, if collisions do contribute to BS formation, they should occur with the greatest frequency in high density clusters.

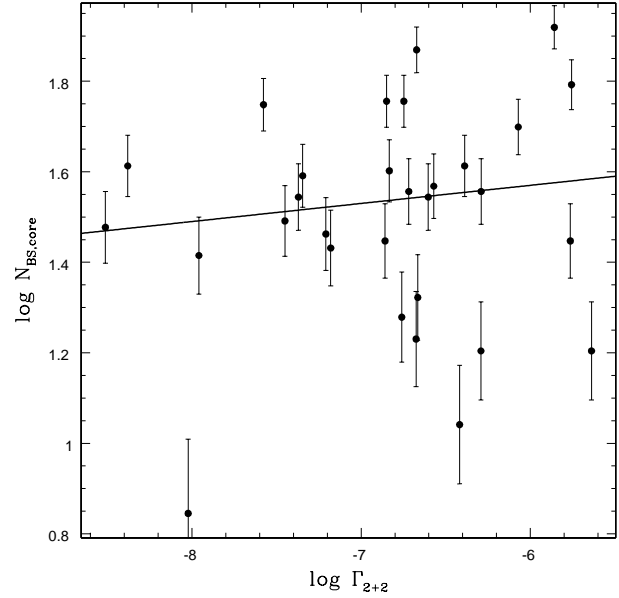
For both sub-samples of low- and high-density clusters, our results remain consistent with what we found for the entire sample to within the uncertainties. This is supported both by our lines of best-fit and our Spearman correlation coefficients. Therefore, this is consistent with the general picture that the dominant BS formation mechanism is the same in all clusters (as opposed to different mechanisms dominating in different clusters).

Next, we search for a linear combination of the different formation channels that yields a better correlation with the observed BS numbers than any of these parameters individually. This is done in two ways. Specifically, we fit to the observed data relations of the form:

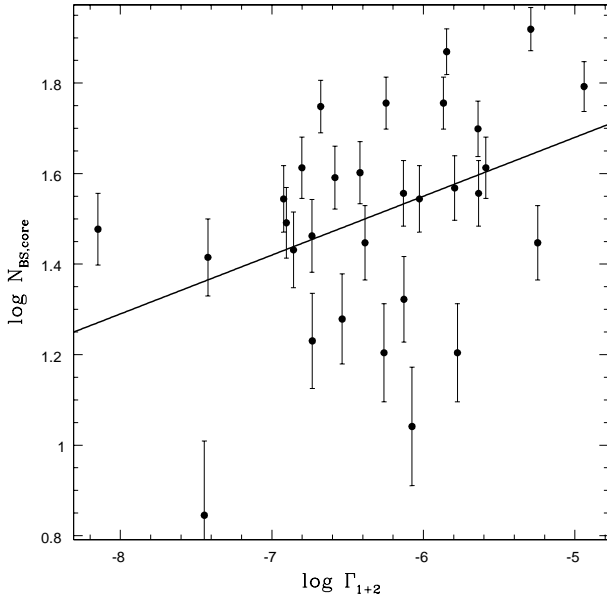
$$N_{BS} = aN_{bin,core} + bN_{1+1} + cN_{1+2} + dN_{2+2}, \quad (8)$$



**Figure 6.** The logarithm of the number of BSs as a function of the single-single collision rate in the core (in units of number of collisions per year). The solid lines shows the best-fit to the data.



**Figure 8.** The logarithm of the number of BSs as a function of the binary-binary collision rate in the core (in units of number of collisions per year). The solid line shows the best-fit to the data.



**Figure 7.** The logarithm of the number of BSs as a function of the single-binary collision rate in the core (in units of number of collisions per year). The solid line shows the best-fit to the data.

and

$$N_{BS} = e f_{bin,core}^f M_{core}^g + h f_{bin,core}^i N_{1+1}^j, \quad (9)$$

where  $e$ ,  $f$ ,  $g$ ,  $h$ ,  $i$ , and  $j$  are all treated as free parameters, and we omit the factor  $(1 - f_{bin,core})^{-2}$  when calculating  $N_{1+1}$  from Equation 5. These comparisons will help to tell

**Table 1.** Slopes for all weighted least-squares fits, and the corresponding Spearman rank correlation coefficients and probabilities that the null hypothesis of zero correlation is disproved.

Parameter	$\log N_{BS}$ slope; $r_s$ ; $p_s$
$\log N_{bin,core}$	$0.48 \pm 0.09$ ; 0.61; $3.80\text{e-}4$
$\log N_{bin,half}$	$0.53 \pm 0.11$ ; 0.49; $5.91\text{e-}3$
$\log \Gamma_{1+1}$	$0.16 \pm 0.04$ ; 0.60; $5.14\text{e-}4$
$\log \Gamma_{1+2}$	$0.13 \pm 0.06$ ; 0.36; $4.86\text{e-}2$
$\log \Gamma_{2+2}$	$0.04 \pm 0.06$ ; 0.06; $7.65\text{e-}1$
$\log M_{core}$	$0.40 \pm 0.05$ ; 0.83; $1.57\text{e-}8$

us if multiple formation mechanisms contribute significantly to BS formation *in the same cluster*, or if there is always a dominant formation channel.

Equation 8 is unable to provide a more statistically significant correlation than we find between BS numbers and the core masses. The best-fitting model offers at best a slight improvement over what we find upon comparing BS population size to either the number of binaries in the core or the 1+1 collision rate alone. Finally, the only best-fitting parameter in Equation 9 we find to be consistent with a non-zero value is the power-law index on core mass, with  $g = 0.48^{+0.10}_{-0.06}$ . This independently confirms that the strongest dependence we find is between BS numbers and core mass. Therefore, we do not find evidence from this comparison that multiple mechanisms contribute simultaneously to BS formation in *individual* clusters.

## 4 DISCUSSION

### 4.1 Why do the empirical binary fractions fail to yield improved correlations for blue stragglers?

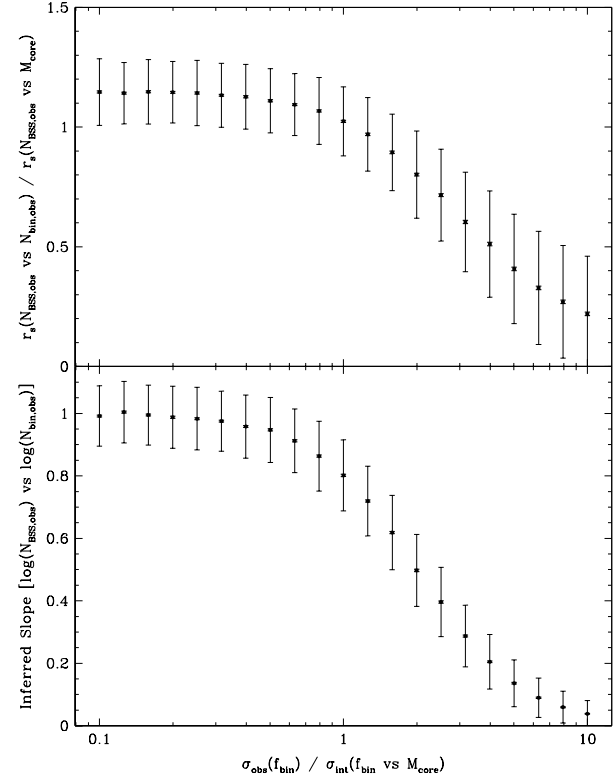
Our main result is that core mass remains a better predictor of blue straggler numbers than any other variable we have considered. This is surprising. In the context of the binary evolution scenario for blue stragglers, one might have expected a stronger correlation with the number of binaries in the core (as estimated by  $f_{bin,core} \times M_{core}$ ). Similarly, in the context of the collision scenario, it seems reasonable to suppose that blue stragglers may form primarily in collisions involving binaries. In this case, stronger correlations with either 1+2 or 2+2 collision rates might be expected. Yet neither of these hypotheses is confirmed by the data.

At first sight, this negative result is all the more surprising because the binary fractions themselves *do* (anti-)correlate strongly with core mass (and absolute cluster mass). As noted in Knigge, Leigh & Sills (2009), this anti-correlation is exactly what is needed to explain the sub-linear correlation between BS numbers and core mass in the binary evolution model. So why does the combination of core mass and binary fractions not lead to a roughly linear correlation between binary and BS numbers, as one might naively expect?

It is, of course, possible that the core mass correlation is simply the most fundamental one. However, if this correlation is not driven by binaries, its origin and sub-linear nature are quite hard to understand.

As it turns out, there is, in fact, a simple way to understand our results in the context of the binary evolution model. The easiest way to see this is to recognize that the existence of a correlation between  $M_{core}$  and  $f_{bin,core}$  (Figure 3) obviously implies that core mass is itself an estimator of core binary fractions. Thus even if  $N_{BS}$  depends solely on  $N_{bin,core}$ , replacing  $M_{core}$  with  $f_{bin,core} \times M_{core}$  will only lead to an improved correlation if the observational errors on the empirical binary fractions are smaller than the intrinsic scatter in the  $f_{bin,core}$  versus  $M_{core}$  relationship.

In order to illustrate this quantitatively, we have carried out some simple simulations. Thus we create mock data sets containing  $N = 30$  data points and spanning roughly the same dynamical range as the real data. In each mock data set, we assume that the true number of BSs scales perfectly and linearly with the number of binaries. We also assume that binary fractions correlate sub-linearly with core masses,  $f_{bin,core} \propto M_{core}^{-0.6}$ , and that this correlation is quite tight, with an intrinsic dispersion of  $\sigma_{int} = 0.1$  dex. We also assume that BS numbers are subject to an observational error of 0.1 dex, and, for simplicity, that core masses are perfectly known. Finally, we assume that the binary fractions are subject to observational uncertainties,  $\sigma_{obs}(f_{bin,core})$ . We are interested in how the character of observationally inferred correlations changes when  $\sigma_{obs}(f_{bin,core})$  approaches and exceeds  $\sigma_{int}$ , so we run tests over the range  $0.1\sigma_{int} \leq \sigma_{obs}(f_{bin,core}) \leq 10.0\sigma_{int}$ . For each trial value of  $\sigma_{obs}(f_{bin,core})$ , we create 1000 mock data sets and measure the Spearman-rank correlation coefficients of the  $N_{BS,obs}$  versus  $M_{core}$  relation and the  $N_{BS,obs}$  versus  $N_{bin,obs}$  relation. We also measure the slope of the  $\log N_{BS,obs}$  versus  $\log N_{bin,obs}$  correlation in each mock data set, in order to



**Figure 9.** The results of our simulations to quantify whether the observational errors on the empirical binary fractions are smaller than the intrinsic scatter in the  $f_{bin,core}$  versus  $M_{core}$  relation. Our procedure for this has been described in detail in the text. Note that the error bars shown in this plot correspond to the standard deviation across the mock samples, as opposed to the error for the mean.

check if and when this deviates substantially from the true slope of unity.

The results of the simulations are shown in Figure 9. They confirm that the correlation coefficient of the  $N_{BS,obs}$  versus  $N_{bin,obs}$  relation exceeds that of the  $N_{BS,obs}$  versus  $M_{core}$  relation only if  $\sigma_{obs}(f_{bin,core}) \lesssim \sigma_{int}$ . Once the errors on  $f_{bin,core}$  exceed this, the correlation with core mass becomes stronger than that with  $N_{bin,obs}$ , as in the actual data. It is also interesting that, in the same regime, the measured slope of the  $\log N_{BS,obs}$  versus  $\log N_{bin,obs}$  relation becomes significantly shallower than the true slope of unity. Again, this matches what we see in our analysis of the actual data.

We therefore suggest that the simplest way to understand our results is to assume that the observational uncertainties on the empirical binary fractions exceed the intrinsic dispersion in the  $f_{bin,core}$  versus  $M_{core}$  relationship. If this is correct, the observational data can be understood in the context of the simple binary evolution model. It is worth stressing here that we are not suggesting that the observed binary fractions are “wrong” – merely that the uncertainties affecting them are larger than  $\sigma_{int}$ . This, in turn, is astrophysically important. The observed anti-correlation between  $f_{bin,core}$  and core mass (Figure 3) or total cluster mass (Milone et al. (2012)) is already surprising and should

be a key benchmark for dynamical models of GCs. If our suggestion here is correct, this correlation is even tighter than the present data suggest.

Having suggested our preferred explanation for our initially surprising results, we will devote the rest of this section to explore the viability of other possibilities. In particular, we will carefully consider the potential impact of key assumptions in our analysis.

## 4.2 A check on key assumptions in the analysis

### 4.2.1 Constant average stellar mass

We have assumed throughout our analysis that the average stellar mass,  $\bar{m}$ , is constant across all clusters. In reality, the average main-sequence mass can range from  $\sim 0.3 M_{\odot}$  to  $\sim 0.7 M_{\odot}$ , and there seems to be a connection between mass function slope and the total GC mass, with lower-mass GCs being more depleted of preferentially low-mass stars (Leigh et al. 2012). This suggests that the average stellar mass should increase with decreasing cluster mass. However, replacing the assumption of a constant average mass in Equation 3 with a dependence of the form  $\bar{m} \propto M_{core}^{\epsilon}$ , with  $\epsilon < 0$ , should further *flatten* the dependence of BS numbers on the calculated numbers of binaries. This suggests that our assumption for the average stellar mass is not the cause of the observed non-linear dependence of BS numbers on the numbers of binaries.

We adopt the same assumption of a constant  $\bar{m}$  when calculating the 1+1, 1+2, and 2+2 collision rates. The power-law indices we find with BS numbers for all three of these parameters are very small ( $\approx 0.1$ ), and the range in average stellar masses for our sample is at most  $\approx 0.4 M_{\odot}$ . Therefore, there is no realistic assumption for the average stellar mass that we could adopt to recover a linear relation between BS population size and any of the collision rates.

### 4.2.2 Constant average semi-major axis for binaries

Similarly, we assumed a constant value of  $\bar{a} = 2$  AU for the average semi-major axes of all binaries undergoing 1+2 and 2+2 encounters. However, it is possible that  $\bar{a}$  depends systematically on the cluster mass, since the semi-major axis corresponding to the hard-soft boundary depends on the cluster mass (via the velocity dispersion). We replaced  $\bar{a} = 2$  AU in our estimates for the 1+2 and 2+2 collision rates with the semi-major axis corresponding to the hard-soft boundary in each cluster, and re-performed our analysis for these two parameters. This did not improve the agreement between the observed BS numbers and either the 1+2 or 2+2 collision rates.

### 4.2.3 Other cluster-to-cluster variations

There are a number of ways that cluster-to-cluster variations in the distributions of binary orbital parameters could have affected our results. For instance, there are several reasons why high-mass MS-MS binaries with mass ratios  $q \sim 1$  are the most likely to produce BSs. It is MS stars with masses just below that of the turn-off that are next in line to ascend the giant branch. Provided they are in binaries, they are therefore the best candidates to over-fill their Roche

lobes within the next few hundred Myrs. Two MS stars with masses close to the turn-off should also produce the brightest and bluest BSs upon merging (e.g. Sills et al. 2001), either via binary coalescence or collisions. These are the most likely to stand out as BSs in the cluster colour-magnitude diagram.

The distribution of binary orbital separations should affect not only the frequency of mass-transfer events, but also the outcomes of dynamical interactions involving binaries. It is the closest binaries that are the most likely to tidally interact and undergo mass-transfer (e.g. Mathieu & Geller 2009), and the probability of a collision occurring during 1+2 and 2+2 interactions increases with decreasing binary semi-major axis (Fregeau et al. 2004). Similar arguments can also be extended to orbital eccentricity distributions that are richer in high-eccentricity orbits. All of this suggests that the dependences of the various binary parameter distributions on total cluster mass could, in principle, play a role in driving the observed correlations (or lack thereof).

### 4.2.4 The neglect of dynamics

If a given BS currently resides in the cluster core, this does not necessarily mean that it formed there. In particular, many BSs could have either formed outside the core before migrating in due to dynamical friction, or they could have formed inside the core from binary progenitors that recently migrated in (e.g. Mapelli et al. 2006). We found in Leigh, Sills & Knigge (2011c) that the observed dependence of BS numbers on core mass can only be reproduced if at least some BSs did indeed recently migrate in from outside the core. Specifically, we found that this contributed to lowering the power-law index on  $M_{core}$ . This is because the time-scale for two-body relaxation increases with increasing cluster mass, so that fewer BSs formed outside the cores of more massive clusters have had sufficient time to migrate in via dynamical friction. If correct, this predicts that the *global* numbers of BSs should correlate more linearly with the *total* cluster mass than we have found for the relation between the numbers of BSs in the *core* and the *core* masses. This can be tested directly using a large sample of cluster CMDs derived using a field of view that extends out to the tidal radius in all clusters. The on-going work of, for example, Fekadu, Sandquist & Bolte (2007), Dalessandro et al. (2009), Carraro & Selezney (2011), Beccari et al. (2011), and Sanna et al. (2012) should prove very useful in this regard. This is because they have slowly been compiling a large sample of CMDs with nearly complete spatial coverage, and it will be possible to compile from this a homogeneous sample of cluster CMDs for which the field of view consistently includes the entire cluster.

On the other hand, most normal binaries currently populating the core are likely to have spent a significant fraction of their lives there (e.g. Heggie & Hut 2003). This means that they have had plenty of time to have been affected by the cluster dynamics. Therefore, any BSs currently in the core that recently migrated in from the cluster outskirts were formed from a more primordial component of the total binary population, whereas the present-day core binary fractions reflect a more dynamically-processed component. The effects of this could include a weakening of the correlation between the observed number of BSs in the core and



the *present-day* number of binaries in the core. However, it would *not* affect an underlying correlation with the cluster or core mass. This would be consistent with the data, so this effect may contribute to our results.

#### 4.2.5 Selection effects

One final effect worth considering is the reliance of our BS selection criteria on location in the colour-magnitude diagram. In particular, we find that the number of BSs scales sub-linearly with the core mass. Therefore, in order for some issue with our CMD-based selection criteria to explain our results, we require that proportionately fewer BSs appear considerably brighter and bluer than the main-sequence turn-off in massive cluster cores when compared to low-mass cores. This could arise if, for example, more merger and mass-transfer products appear hidden along the MS in preferentially massive clusters, instead of appearing distinctly brighter and bluer than the MSTO. If correct, this predicts that the average BS luminosity should decrease with increasing core mass. It follows that the average BS mass should also decrease with increasing core mass, since previous studies have shown that the luminosities of BSs are correlated with their masses (e.g. Sills et al. 2001). This offers a useful test of the idea that the number of “blue stragglers” hidden along the MS in the CMD depends systematically on the total cluster mass.

## 5 SUMMARY

We have carried out a statistical analysis to study the origins of blue stragglers in a large sample of Galactic globular clusters, based on data obtained as part of the ACS Survey for Globular Clusters. The main novel ingredient in our analysis are empirically estimated core binary fractions, which allow us to estimate the number of core binaries, as well as the 1+2 and 2+2 collision rates. Contrary to our expectations, we have found that none of these observationally estimated quantities yield correlations with BS numbers that improve upon the previously known sub-linear correlation between BS numbers and core mass. This is despite the fact that the binary fractions themselves anti-correlate strongly with core mass, just as expected in a simple binary evolution model, where the number of BSs would scale linearly with the number of binaries.

We have explored several possible explanations for our results. The simplest, and most appealing, is that observational uncertainties affecting the core binary fractions exceed the intrinsic scatter of the relationship between binary fractions and core mass. This could reconcile the data with the binary evolution model. In the context of the binary evolution model, this would explain why the product of binary fraction and core mass is a poorer predictor of BS numbers than core mass alone, and also why the relationship between the observed numbers of binaries and BS numbers is sub-linear.

If this explanation is correct, it would imply that core binary fractions are tightly coupled to the core or total cluster mass. This would be of considerable significance for the dynamical evolution of globular clusters, and provides an

important benchmark for simulations attempting to understand their present-day properties.

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